Modeling and Analysis of Guided-Wave Structures Involving Both Bi-Isotropic and Bi-Anisotropic Media

Jifu HUANG and Ke WU

POLY-GRAMES Research Center
Dept. de génie électrique et de génie informatique
École Polytechnique, C. P. 6079, Succ. Centre-Ville
Montréal, Canada H3C 3A7

Abstract A new strategy of modeling and analysis with transmission line matrix (TLM) algorithm is developed to account for dynamic effects of both bi-isotropic and bi-anisotropic media on propagation and scattering characteristics. First, the symmetrical condensed node in the frequency-domain is generalized to involving bi-anisotropic media. The nodal scattering matrix is derived directly from Maxwell's equations using the centered finite difference and the transformation of variable. With the proposed node, a frequency-domain TLM algorithm is then established for analysis of propagation and scattering of arbitrary waveguiding structures including discontinuities such as chiral-filled rectangular waveguide, microstrip on a bi-isotropic non-reciprocal or chiral substrate. It is shown that the proposed TLM modeling provides a powerful tool for theoretical study of a new class of complex materials.

Introduction

The study of electromagnetic wave propagation along bi-isotropic and bi-anisotropic media is very important since they offer some promising applications in microwave technology as well as radio engineering. The interesting properties of these materials have been intensively investigated in recent years [1]-[8]. However, in spite of numerous works published on this subject, only very few cases have been reported that are essentially solved by numerical techniques. The analytical methods can only be applied to some special limited circumstances. Therefore, it is imperative to develop an efficient mathematical formalism for unified analysis of these new transmission media.

In the present work, we propose a new modeling strategy using the frequency-domain transmission line matrix (FDTLM) method in which the algorithm of [9] is generalized from isotropic (anisotropic) to bi-isotropic (bi-anisotropic) media. These materials are first characterized by linear constitutive relations that couple the electric and the magnetic field vectors by four independent tensors ($\vec{\vec{\varepsilon}}$, $\vec{\vec{\mu}}$, $\vec{\vec{\xi}}$, $\vec{\vec{\zeta}}$). The electromagnetic field components of Maxwell's equations are then transformed into nodal voltages and loop currents of TLM framework with reference to Johns' symmetrical condensed node [10]. Combining a centered-finite difference of network differential equations and a network parameter transformation leads to a FDTLM nodal scattering matrix. Finally, a TLM algorithm [9] based on the proposed nodal scattering matrix is implemented in the frequency-domain to calculate 2D eigenvalue and 3D discontinuity problems of waveguiding structures. A number of numerical examples are presented to verify the proposed theory and also demonstrate some interesting properties of bi-isotropic and bi-anisotropic media.

Theory

The constitutive relations in a most general bi-anisotropic medium can be written (assuming the $exp(j\omega t)$ time-dependence) in the form of

$$\vec{D} = \vec{\vec{\varepsilon}}\vec{E} + \vec{\vec{\xi}}\vec{H}$$
$$\vec{B} = \vec{\vec{\mu}}\vec{H} + \vec{\vec{\zeta}}\vec{E} \qquad (1)$$

where

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$$\vec{\varepsilon}=\varepsilon_0\begin{pmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}\end{pmatrix} \qquad \vec{\mu}=\mu_0\begin{pmatrix}\mu_{xx} & \mu_{xy} & \mu_{xz}\\ \mu_{yx} & \mu_{yy} & \mu_{yz}\\ \mu_{zx} & \mu_{zy} & \mu_{zz}\end{pmatrix}$$

and

$$\vec{\xi}=\begin{pmatrix}\xi_{xx} & \xi_{xy} & \xi_{xz}\\ \xi_{yx} & \xi_{yy} & \xi_{yz}\\ \xi_{zx} & \xi_{zy} & \xi_{zz}\end{pmatrix} \qquad \vec{\zeta}=\begin{pmatrix}\zeta_{xx} & \zeta_{xy} & \zeta_{xz}\\ \zeta_{yx} & \zeta_{yy} & \zeta_{yz}\\ \zeta_{zx} & \zeta_{zy} & \zeta_{zz}\end{pmatrix}$$

$\vec{\varepsilon}$ and $\vec{\mu}$ are permittivity tensor and permeability tensor of the medium, respectively, and the tensors $\vec{\xi}$ and $\vec{\zeta}$ characterize the coupling between electric and magnetic fields. Using (1) together with Maxwell's equation, we have

$$\begin{aligned}
\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z} &= j\omega\varepsilon_0\left(\varepsilon_{xx}E_x+\varepsilon_{xy}E_y+\varepsilon_{xz}E_z\right)+j\omega\left(\xi_{xx}H_x+\xi_{xy}H_y+\xi_{xz}H_z\right)\\
\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x} &= j\omega\varepsilon_0\left(\varepsilon_{yx}E_x+\varepsilon_{yy}E_y+\varepsilon_{yz}E_z\right)+j\omega\left(\xi_{yx}H_x+\xi_{yy}H_y+\xi_{yz}H_z\right)\\
\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y} &= j\omega\varepsilon_0\left(\varepsilon_{zx}E_x+\varepsilon_{zy}E_y+\varepsilon_{zz}E_z\right)+j\omega\left(\xi_{zx}H_x+\xi_{zy}H_y+\xi_{zz}H_z\right)\\
\frac{\partial E_y}{\partial z}-\frac{\partial E_z}{\partial y} &= j\omega\mu_0\left(\mu_{xx}H_x+\mu_{xy}H_y+\mu_{xz}H_z\right)+j\omega\left(\zeta_{xx}E_x+\zeta_{xy}E_y+\zeta_{xz}E_z\right)\\
\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z} &= j\omega\mu_0\left(\mu_{yx}H_x+\mu_{yy}H_y+\mu_{yz}H_z\right)+j\omega\left(\zeta_{yx}E_x+\zeta_{yy}E_y+\zeta_{yz}E_z\right)\\
\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x} &= j\omega\mu_0\left(\mu_{zx}H_x+\mu_{zy}H_y+\mu_{zz}H_z\right)+j\omega\left(\zeta_{zx}E_x+\zeta_{zy}E_y+\zeta_{zz}E_z\right)
\end{aligned} \tag{2}$$

To establish a standard TLM formulation based on Johns' symmetrical condensed node [10], the following equality is established that relates network voltages and currents to the electric and magnetic fields:

$$\begin{aligned}
&V_x=\Delta x\cdot E_x\,,\ V_y=\Delta y\cdot E_y\,,\ V_z=\Delta z\cdot E_z\\
&I_x=\Delta x\cdot Z_0\cdot H_x\,,\ I_y=\Delta y\cdot Z_0\cdot H_y\,,\ I_z=\Delta z\cdot Z_0\cdot H_z\\
&X=x/\Delta x\,,\ Y=y/\Delta y\,,\ Z=z/\Delta z
\end{aligned} \tag{3}$$

where $\Delta x, \Delta y, \Delta z$ are the grid dimensions and Z_0 is the characteristic impedance of free space. Substituting (3) into (2) leads to a set of coupled nodal voltage and current differential equations

$$\begin{aligned}
\frac{\partial I_z}{\partial Y}-\frac{\partial I_y}{\partial Z} &= \left(Y_{xx}V_x+Y_{xy}V_y+Y_{xz}V_z\right)+\left(C_{xx}I_x+C_{xy}I_y+C_{xz}I_z\right)\\
\frac{\partial I_x}{\partial Z}-\frac{\partial I_z}{\partial X} &= \left(Y_{yx}V_x+Y_{yy}V_y+Y_{yz}V_z\right)+\left(C_{yx}I_x+C_{yy}I_y+C_{yz}I_z\right)\\
\frac{\partial I_y}{\partial X}-\frac{\partial I_x}{\partial Y} &= \left(Y_{zx}V_x+Y_{zy}V_y+Y_{zz}V_z\right)+\left(C_{zx}I_x+C_{zy}I_y+C_{zz}I_z\right)\\
\frac{\partial V_y}{\partial Z}-\frac{\partial V_z}{\partial Y} &= \left(Z_{xx}I_x+Z_{xy}I_y+Z_{xz}I_z\right)+\left(L_{xx}V_x+L_{xy}V_y+L_{xz}V_z\right)\\
\frac{\partial V_z}{\partial X}-\frac{\partial V_x}{\partial Z} &= \left(Z_{yx}I_x+Z_{yy}I_y+Z_{yz}I_z\right)+\left(L_{yx}V_x+L_{yy}V_y+L_{yz}V_z\right)\\
\frac{\partial V_x}{\partial Y}-\frac{\partial V_y}{\partial X} &= \left(Z_{zx}I_x+Z_{zy}I_y+Z_{zz}I_z\right)+\left(L_{zx}V_x+L_{zy}V_y+L_{zz}V_z\right)
\end{aligned} \tag{4}$$

In the network model of TLM, by using a centered-finite difference at the center of symmetrical condensed node, a set of voltage and current components are obtained at the boundary planes of the node. Each pair of voltage and current variables consists of a polarized planar wave. Thereafter, the voltage and current variables are adequately transformed at the nodal boundary planes into relevant incident and reflected waves. After some manipulation, a relationship can be set up to interrelate the nodal voltage and current variables to the corresponding incident voltages as described in the following matrix equation:

$$\begin{pmatrix}
Y_x & Y_{xy} & Y_{xz} & C_{xx} & C_{xy} & C_{xz}\\
Y_{yx} & Y_y & Y_{yz} & C_{yx} & C_{yy} & C_{yz}\\
Y_{zx} & Y_{zy} & Y_z & C_{zx} & C_{zy} & C_{zz}\\
L_{xx} & L_{xy} & L_{xz} & Z_x & Z_{xy} & Z_{xz}\\
L_{yx} & L_{yy} & L_{yz} & Z_{yx} & Z_y & Z_{yz}\\
L_{zx} & L_{zy} & L_{zz} & Z_{zx} & Z_{zy} & Z_z
\end{pmatrix}
\begin{pmatrix}V_x\\ V_y\\ V_z\\ I_x\\ I_y\\ I_z\end{pmatrix}
=
\begin{pmatrix}\Psi_x\\ \Psi_y\\ \Psi_z\\ \Phi_x\\ \Phi_y\\ \Phi_z\end{pmatrix} \tag{5}$$

where

$\Psi_x=2\left(V_2^i+V_9^i+V_1^i+V_{12}^i\right)$, $\Psi_y=2\left(V_4^i+V_8^i+V_3^i+V_{11}^i\right)$,

$\Psi_z=2\left(V_5^i+V_7^i+V_6^i+V_{10}^i\right)$, $\Phi_x=2\left(V_5^i+V_8^i-V_4^i-V_7^i\right)$,

$\Phi_y=2\left(V_2^i+V_{10}^i-V_6^i-V_9^i\right)$, $\Phi_z=2\left(V_3^i+V_{12}^i-V_1^i-V_{11}^i\right)$.

The immediate step is now to make appropriate averaging of the relevant nodal voltages and currents at the center of the node. It yields six pairs of hybrid equations that interrelate reflected and incident voltages. For example, for ports 1 and 12, we have

$$\begin{aligned}
V_1^r &= V_x+I_z-V_{12}^i\\
V_{12}^r &= V_x-I_z-V_1^i
\end{aligned} \tag{6}$$

The combination of (5) with (6) leads to a full 12×12 nodal scattering matrix that completely describes the scattering property of the frequency-domain TLM node.

Numerical Examples and Discussion

To examine computational feature and validity of the proposed node, a TLM algorithm [9] based on this nodal scattering matrix is implemented in the frequency-domain. Our particular interest is to apply the present theory to a class of bi-isotropic and bi-anisotropic waveguiding structures.

As the first example, a microstrip structure on bi-isotropic substrate is considered, which was also studied in [6, 7]. In this case, the substrate consists of a pure non-reciprocal material with Tellegen parameter χ or a pure reciprocal material with chirality parameter κ. Therefore, the tensors $\overleftrightarrow{\xi}$ and $\overleftrightarrow{\zeta}$ can be reduced to scalar quantities, such that,

$$\begin{aligned} \xi &= (\chi - j\kappa)\sqrt{\mu_0 \varepsilon_0} \\ \zeta &= (\chi + j\kappa)\sqrt{\mu_0 \varepsilon_0} \end{aligned} \quad (7)$$

Fig. 1 shows a comparison of our results with the quasi-static solution available in [6, 7]. The normalized phase velocities are obtained as a function of Tellegen parameter χ or chirality parameter κ. It is clear that the results of the proposed analysis tends to converge towards that of the quasi-static approximations (dashed lines) as the operating frequency decreases. This indicates that a very good agreement can be expected for the complete static situation. In our second example, a bi-isotropic material is asymmetrically inserted longitudinally in a rectangular waveguide. An interesting non-reciprocity is observed in Fig. 2 for the magneto-electric effect of parameter χ. Consequently, we can explore a possibility of using bi-isotropic materials instead of (or combined with) magnetically biased ferrite in the design of novel devices and components.

To look further into the described method, scattering characteristics are calculated for a rectangular waveguide partially loaded with generalized bi-isotropic material involving both parameters χ and κ (Fig. 3). The magnitudes and phases of the transmission and reflected coefficients of the TE_{10}-wave are determined as a function of frequency. It is found that χ leads to the non-reciprocal phase shifter ($\Phi_{12} \neq \Phi_{21}$) while κ results in the chirality ($\Phi_{11} \neq \Phi_{22}$). It should be noted that, only the analysis of rectangular waveguide discontinuities is concerned in the present paper, such a method can be readily extended to modeling of arbitrary guided-wave structures.

Conclusions

In this work, a new FDTLM modeling has been generalized for considering both bi-isotropic and bi-anisotropic media in guided-wave structures. An efficient and accurate TLM algorithm using the proposed TLM node is developed and used to the study of a class of complex structures. A number of numerical examples are presented to verify the present theory and also to demonstrate its usefulness and generality. It is shown that interesting property can be derived from using these particular materials in the guided-wave structures. It comes to conclude that the present field-theoretical modeling technique may pave the way for establishing a unified analysis and design tool for a new family of devices and circuits.

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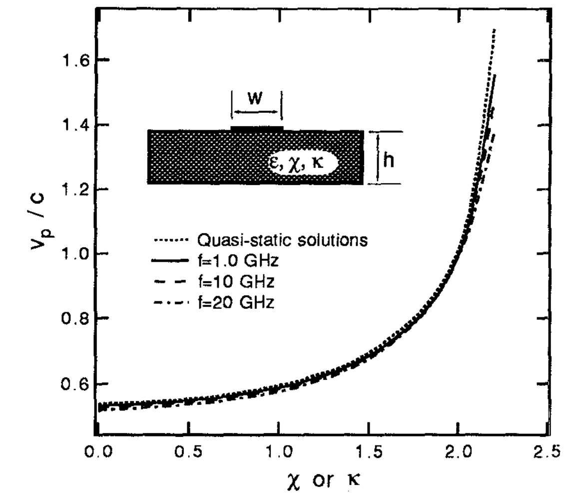

Fig. 1 Normalized phase velocity as a function of χ or κ for microstrip geometry of ε_r =5, and w/h =1.

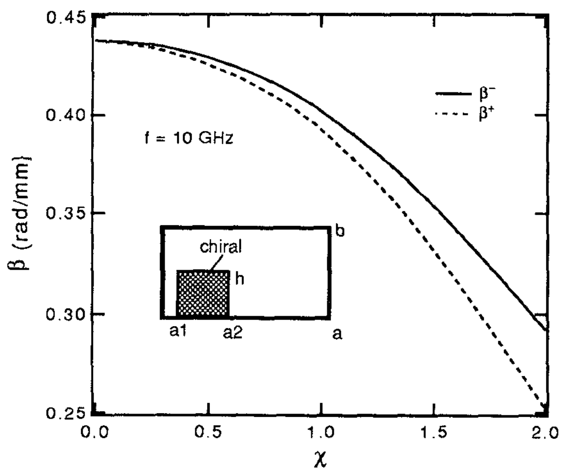

Fig. 2 Phase constants of the fundmantal mode of a rectangular waveguide partially filled with chiral slab (a = 22.86 mm, b = 10.16 mm, a1= a/10, a2= 3a/10, h = b/2).

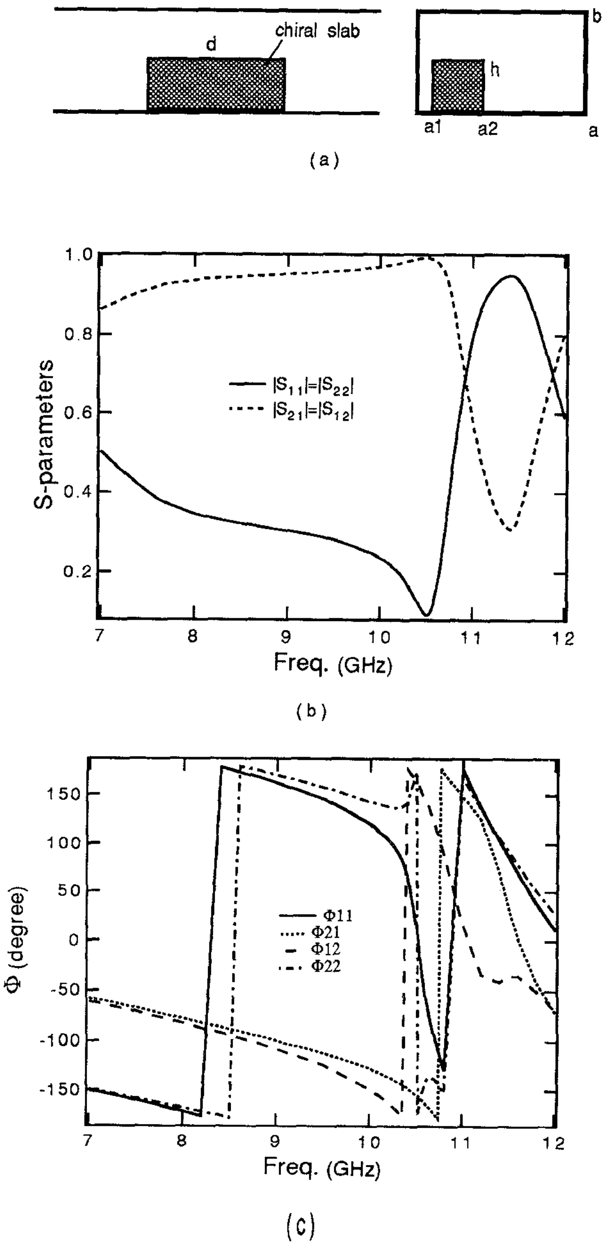

Fig. 3 (a) A asymmetrically placed bi-isotropic slab in rectangular waveguide (a=22.86 mm, b=10.16 mm, a1=a/10, a2=3a/10, h=b/2, d=10 mm, chiral slab parameters: χ=1.0, κ=0.5, ε_r=9.0);
(b) magnitudes of S-parameters; (c) Phases of S-parameters.